Key

Math 4

4-2 Law of Sines and Cosines

Name________________________

Date________

Learning goal:

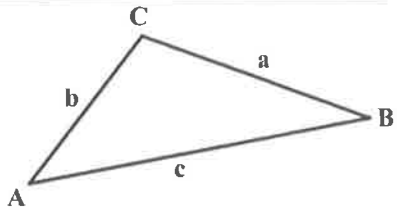
I can find missing sides and angles from non-right triangles using the Law of Sines and Cosines.

In Math 2 and Math 3, you learned how to find missing sides and angles of triangles using sine, cosine and tangent. What happens if you are missing a side or angle from a non-right triangle? This is why we have the Law of Sines and the Law of Cosines.

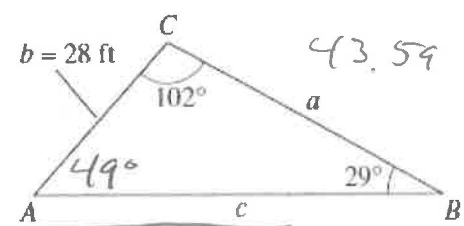
Law of Sines

If ABC is a triangle with sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$



1. Use the Law of Sines to find the remaining angle and sides lengths of the figure below.



$180-102-29=49°$

$$\frac{\sin 29}{28} = \frac{\sin 49}{a}$$

$a \cdot \sin 29 = 28 \cdot \sin 49$

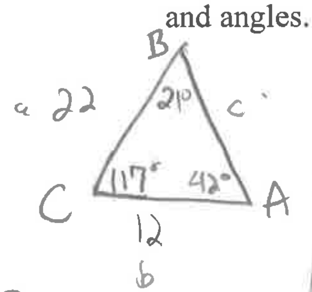
$a = \dfrac{28 \cdot \sin 49}{\sin 29} \approx \boxed{43.59 \text{ ft}}$

$$\frac{c}{\sin 102} = \frac{28}{\sin 29}$$

$c = \dfrac{28 \cdot \sin 102}{\sin 29}$

$\boxed{c \approx 56.49 \text{ ft}}$

2. Suppose ΔABC exists such that $a = 22$ inches, $b = 12$ inches and $A = 42°$. Find the remaining side and angles.



*Picture not to scale!

$$\frac{22}{\sin 42} = \frac{12}{\sin B}$$

$\sin B = \dfrac{12 \cdot \sin 42}{22}$

$\sin B \approx 0.365$

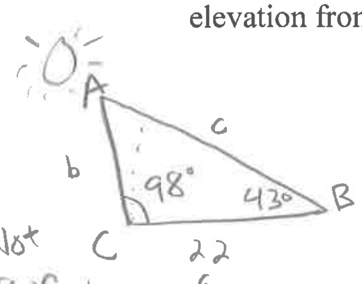
$B \approx \boxed{21°}$

$\angle C = 180-21-42 = 117°$

$$\frac{c}{\sin 117} = \frac{22}{\sin 42}$$

$c = \dfrac{22 \cdot \sin 117°}{\sin 42°} \approx \boxed{29.29 \text{ in}}$

3. A pole tilts toward the sun at an $8°$ angle from vertical, and it casts a 22-foot shadow. The angle of elevation from the tip of the shadow to the top of the pole is $43°$. How tall is the pole?



*Not drawn to scale.

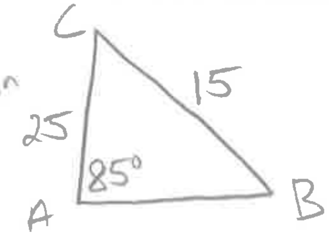
$$\frac{b}{\sin 43} = \frac{22}{\sin 39}$$

$\angle A = 39°$

$b = \dfrac{22 \cdot \sin 43}{\sin 39}$

$\boxed{b \approx 23.84 \text{ ft}}$

4. Find the missing side length and angles for ΔABC if $a = 15$, $b = 25$ and $A = 85°$.



*Not drawn to scale

$$\frac{25}{\sin B} = \frac{15}{\sin 85}$$

$\sin B = \dfrac{25 \cdot \sin 85}{15}$

$\sin B = 1.66$

$\boxed{\text{No solution}}$

No triangle exists with those measurements!

Hooray! Now we can find missing parts of triangles that aren't right triangles. I'm sure you feel like your life is complete. Now take a look at the two figures below.

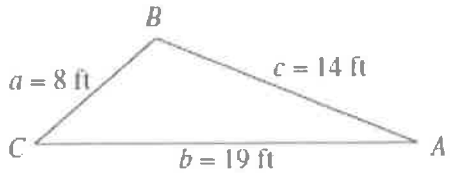


Figure 1

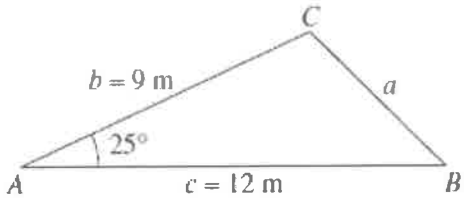


Figure 2

If I asked you to find angle A in Figure 1, how would you do it? What about side a in Figure 2? Unfortunately, Law of Sines is not going to help us here. Don't be afraid to cry a little bit. You're only human. If only there was a different Law of something…

Law of Cosines

Standard Form	*Alternative Form*
$a^2 = b^2 + c^2 - 2bc\cos A$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac\cos B$	$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab\cos C$	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

5. Use the Law of Cosines to find the missing angles in Figure 1.

$\cos A = \dfrac{19^2 + 14^2 - 8^2}{2(19)(14)}$

$\cos A = \dfrac{493}{532}$

$\boxed{\angle A \approx 22°}$

$\cos B = \dfrac{8^2 + 14^2 - 19^2}{2(8)(14)}$

$\cos B = \dfrac{-101}{224}$

$\boxed{\angle B \approx 117°}$

$\angle C = 180 - 117 - 22$

$\boxed{= 41°}$

6. Use the Law of Cosines to find the missing side and angles in Figure 2.

$a^2 = 9^2 + 12^2 - 2(9)(12)\cos 25°$

$a^2 \approx 29.24$

$\boxed{a \approx 5.41}$

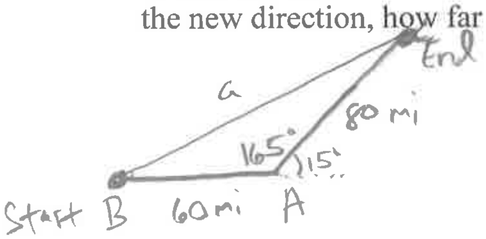
$\cos C = \dfrac{5.41^2 + 9^2 - 12^2}{2(5.41)(9)}$

$\cos C \approx -0.346$

$\boxed{\angle B = 45°}$

$\boxed{\angle C \approx 110°}$

7. A ship travels 60 miles due east, then adjusts its course 15° northward. After traveling 80 miles in the new direction, how far is the ship from its point of departure?



$a^2 = 80^2 + 60^2 - 2(80)(60)\cos 165°$

$a^2 \approx 19{,}272.89$

$\boxed{a \approx 138.83 \text{ miles}}$